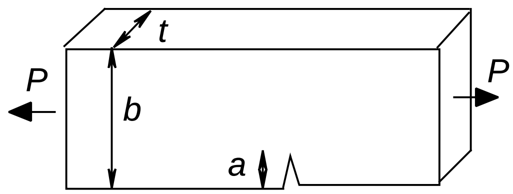
ANOTHER EXAMPLE OF FRACTURE-MECHANICS

Always check for yield and fracture

· Rectangular plate (b= 50 mm x t = 10 mm) with single edge crack (length a)
  Made of aluminum alloy (K_{Ic} = 34 MPa√m, σ_Y = 325 MPa)
  Calculate failure load for different values of a



For this geometry: Remote stress: $\sigma_\infty = P/bt$

In plane of crack: $\sigma_1 \approx \dfrac{P}{(b-a)t}$; $\sigma_2 \approx 0$; $\sigma_3 \approx 0$

(1) Consider yield

· Yield will occur first in plane of crack

Loading parameter for yield: $\tilde{\sigma}_H = \dfrac{P}{(b-a)t}$

· Material property for yield: σ_Y (325 MPa)

· Yield criterion: $\tilde{\sigma}_H \geq \sigma_Y$

Load for yield: $P_Y = \sigma_Y(b - a)t = 325 \times 10^6 (0.050 - a) \times 10^{-2}$

$= 3.25\ (0.050 - a)$ MN (if a in m)

(2) Consider fracture:

· Loading parameter for fracture: $K_I = f(a/b)\sigma_\infty\sqrt{\pi a} = f(a/b)\dfrac{P}{bt}\sqrt{\pi a}$
  where *f(a/b)* given in Dowling or data book

· Material property for fracture: K_{Ic} (34 MPa√m)

· Fracture criterion: $K_I \geq K_{Ic}$

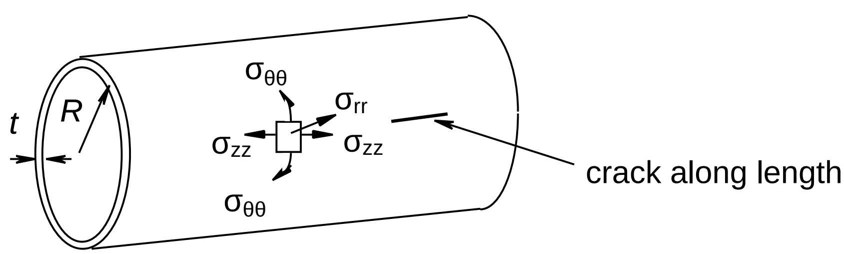
Load for fracture: $P_F = \dfrac{b\ t\ K_{Ic}}{f(a/b)\sqrt{\pi a}} = \dfrac{(5\times 10^{-2})(10^{-2})\times 34\times 10^6}{f(a/b)\sqrt{\pi a}}$

$$= \frac{9.591}{f(a/b)\sqrt{a}}\ \text{kN} \qquad \text{(if } a \text{ in m)}$$

a (m)	*(b-a)* m	*a/b*	*f (a/b)*	P_Y (kN)	P_F (kN)	$P_{Failure}$ (kN)
0	0.050	0	1.12	**163**	∞	160
0.0025	0.0475	0.05	1.16	**154**	165	150
0.005	0.045	0.1	1.25	146	**109**	110
0.010	0.040	0.2	1.37	130	**70.0**	70
0.020	0.030	0.4	2.11	97.5	**32.1**	32

DESIGN OF PRESSURE VESSELS

- Cylindrical pressure vessel: $r >> t$;
- Material parameters: Yield stress: σ_Y; Fracture toughness: K_{Ic}



- Principal stresses: $\sigma_{rr} = -P/2 \approx 0$; $\sigma_{\theta\theta} = PR/t$; $\sigma_{zz} = PR/2t$

(1) Consider yield

- Loading parameter for yield: Von Mises effective stress:

$$\tilde{\sigma}_H \approx \left\{ \frac{1}{2}\left[\left(\frac{PR}{t}\right)^2 + \left(\frac{PR}{2t}\right)^2 + \left(\frac{PR}{2t}\right)^2 \right] \right\}^{1/2} \approx \frac{\sqrt{3}}{2}\frac{PR}{t}$$

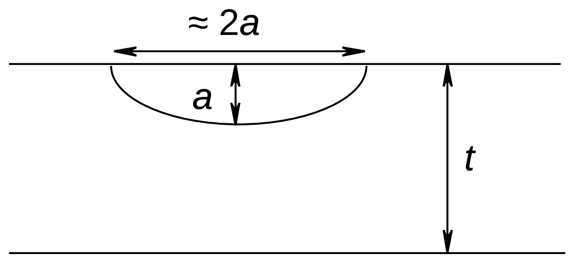
- Material property for yield: σ_Y
- Yield criterion: $\tilde{\sigma}_H \geq \sigma_Y$

Design against yield: $\frac{PR}{t} < \frac{2}{\sqrt{3}}\sigma_Y$ (note: no =!)

(2) Consider fracture:

- Hoop stress, $\sigma_{\theta\theta}$, is largest, therefore axial cracks are most serious

- Pipes and cylinders generally explode by splitting along their length



- Loading parameter for fracture: K_I

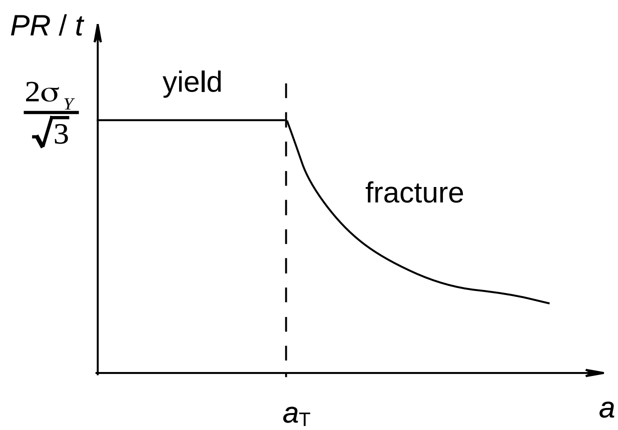
- Can assume that $K_I \approx \sigma_{\theta\theta}\sqrt{\pi a}$

$$K_I = \frac{PR}{t}\sqrt{\pi a}$$

- Material property for fracture: K_{IC}

- Fracture criterion: $K_{\mathrm{I}} \geq K_{\mathrm{Ic}}$

Design against fracture: $\frac{PR}{t} < \frac{K_{Ic}}{\sqrt{\pi a}}$



- Transition crack length, a_T, when

$$\left.\frac{PR}{t}\right|_{\text{yield}} = \left.\frac{PR}{t}\right|_{\text{fracture}}$$

$$\frac{2}{\sqrt{3}}\sigma_Y = \frac{K_{Ic}}{\sqrt{\pi a}}$$

$$a_T \approx \frac{3}{4\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

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Example 1: Pressure-vessel steel ASTM A517-F(K_{Ic} = 187 MPa√m, σ_Y = 760 MPa)

$a_T \approx$ 14 mm

Example 2: 2014 Aluminum alloy (K_{Ic} = 24 MPa√m, σ_Y = 415 MPa)

$a_T \approx$ 0.8 mm

· Much easier to detect a 14 mm long crack than a 1 mm long crack

· Yield is a "safer" failure mechanism than fracture

one design philosophy: make transition crack length as large as possible

· A safer design criterion is "Leak before break"

"Leak-before-break" design for pressure vessels

· Assume cracks are approximately semi-circular

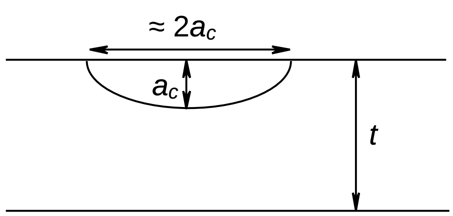
$$K_I \approx \sigma_{\theta\theta}\sqrt{\pi a}\,, \qquad \text{where } \sigma_{\theta\theta} = PR/t$$

Fracture occurs when crack length is given by

$$a = a_c = \frac{1}{\pi}\left(\frac{K_{Ic}t}{Pr}\right)^2$$

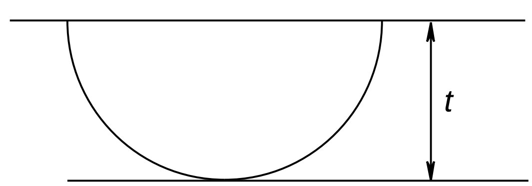
· Two possibilities:

(i) $a_c < t$



Explosion occurs

(ii) $a_c > t$



Pressure vessel will leak before explosion

· Leak-before-break criterion: $a_c > t$

$$\frac{1}{\pi}\left(\frac{K_{Ic}t}{PR}\right)^2 > t$$

$$\frac{PR}{\sqrt{t}} < \frac{K_{Ic}}{\sqrt{\pi}}$$

· Must also remember to design against yield: $\dfrac{PR}{t} < \dfrac{2\sigma_Y}{\sqrt{3}}$

Don't waste time remembering these equations - derive them if needed

Example: Thickness of pressure vessel with P = 1.0 MPa, R = 1.0 m, and safety factor of 3.0 on the thickness.

Use a 2014 Aluminum alloy (K_{Ic} = 24 MPa√m, σ_Y = 415 MPa).

1. Design against yield:

$$\frac{PR}{t} < \frac{2\sigma_Y}{\sqrt{3}}$$

$$t > \frac{\sqrt{3}PR}{2\sigma_Y}$$

$$t > \frac{10^6 \times 1 \times \sqrt{3}}{2 \times 415 \times 10^6} = 3.1 \text{ mm}$$

With safety factor t = 9.3 mm

2. Design for "leak-before -break":

$$\frac{PR}{\sqrt{t}} < \frac{K_{Ic}}{\sqrt{\pi}}$$

$$t > \pi\left(\frac{PR}{K_{Ic}}\right)^2$$

$$t > \pi\left(\frac{10^6 \times 1}{24 \times 10^6}\right)^2$$

$$t > 1.7 \text{ mm}$$

With safety factor t = 5.1 mm

Minimum thickness is 9.3 mm