

Department of Materials Science and Engineering

**DOCTORAL WRITTEN EXAM
January 22, 2015**

Please follow these instructions:

Your exam packet contains 3 questions from each core area for which you signed up, along with several answer sheets. A copy of the Table of Constants is included for your reference. **You must submit 2 questions from each core area you are taking for grading.** You will have 1 1/2 hours to complete each section. You can obtain extra answer sheets from the proctor, if needed. Please use the following procedure:

1. Write a four (4) digit code of your choice, and your name on the page provided. Use this code in place of your name to identify all answer sheets you submit for both days of the exam. Renee will keep the code information, sealed in an envelope, until after the exams are graded.
2. For each answer, use the question sheet as the first page of your answer. If additional pages are required, use the blank answer sheets provided. **At the end of the examination, staple each question sheet and corresponding answer sheets for each question separately**, put this instruction sheet on top of the questions you are turning in, and place them in one side of your exam folder. Place all other exam pages in the other side of your folder, and return everything to Renee if you finish before your time is up.

Please be sure to complete the information required on each page.

Name _____

CODE NUMBER _____

CHECK THE QUESTIONS YOU WISH TO HAVE GRADED:

Materials Physics
And Chemistry:

1. _____

2. _____

3. _____

Advanced Mechanical
Behavior:

4. _____

5. _____

6. _____

Advanced Thermodynamics
Of Materials

7. _____

8. _____

9. _____

Kinetics and Phase
Transformations

10. _____

11. _____

12. _____

Structure of Materials

13. _____

14. _____

15. _____

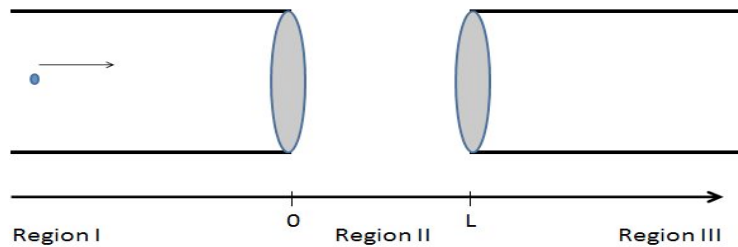
1.

Photoelectric Effect: A sodium photoelectric surface with work function 2.3 eV is illuminated by electromagnetic radiation and emits electrons. The electrons travel toward a negatively charged cathode, completing a circuit. When the potential difference supplied by the power supply is increased to 4.5V, no electrons reach the cathode.

- (a) sketch the experimental set-up to test the photoelectric, labeling the key components.
- (b) for the electrons emitted from the sodium surface, calculate the maximum kinetic energy and the corresponding electron speed.
- (c) calculate the wavelength of the radiation that is incident on the sodium surface
- (d) calculate the minimum frequency of light that will cause photoemission from this sodium surface

2.

Quantum Tunneling: In this problem, we consider the physics of an electron travelling through an air gap in a wire. As shown in the figure below, an electron is initially moving to the right through a very long smooth copper wire with a small air gap in the middle. The work function of copper is V_0 .



(a) For $E > V_0$, sketch the position-dependence of the potential energy of the electron, $V(x)$, with $V=0$ inside the wire. Use a dashed line to show the energy of an electron that satisfies the $E > V_0$ condition. Determine the potential, kinetic, and total energy of the electron both within and outside of the air gap. How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions from high to low. In an adjacent plot, sketch the wave function at $t=0$.

(b) For $E < V_0$, sketch the position-dependence of the potential energy of the electron, $V(x)$, with $V=0$ inside the wire. Use a dashed line to show the energy of an electron that satisfies the $E < V_0$ condition. Determine the potential, kinetic, and total energy of the electron both within and outside of the air gap. How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions from high to low. In an adjacent plot, sketch the wave function at $t=0$.

(c) Discuss the similarities and differences between the wavefunctions and kinetic energies in (a) and (b); for each case, comment on whether or not the electron has enough energy to cross the gap classically.

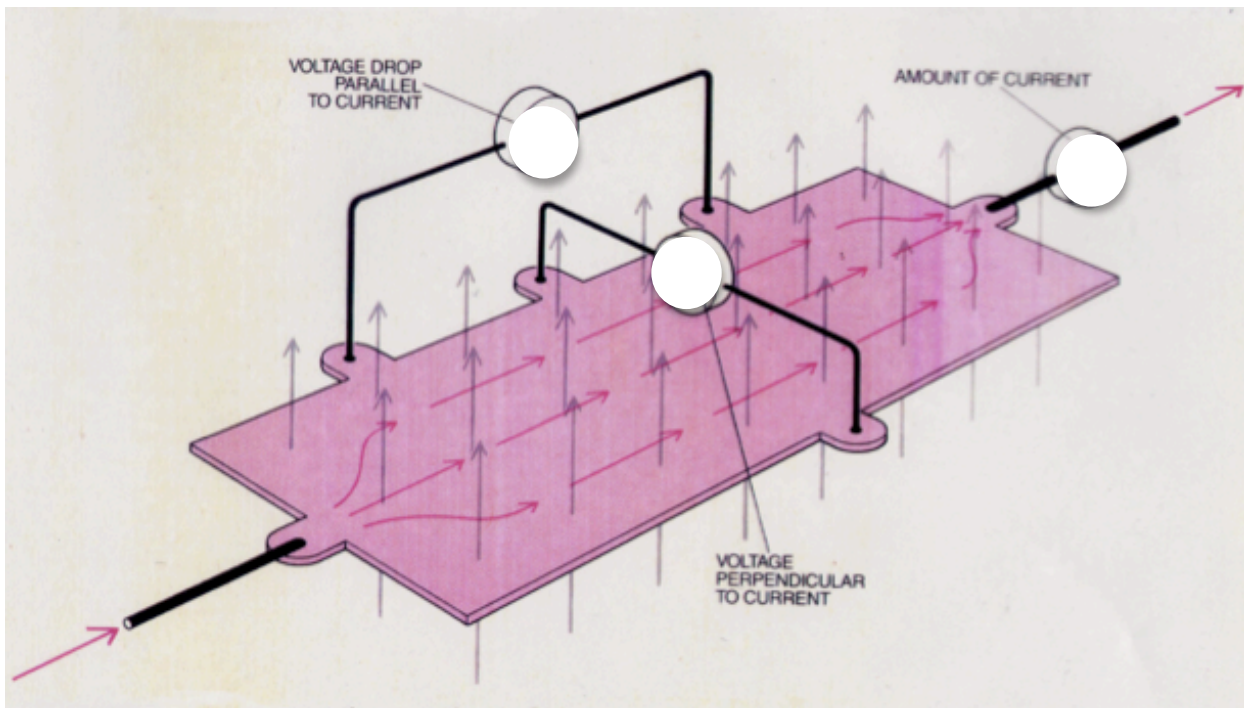
3.

Hall Effect and Fermi Energy

A strip of indium is 1.5 cm wide and 125 μm thick. When a current of 25A is established in the strip and a uniform field of 0.275T is oriented perpendicular to the plane of the strip, a Hall voltage of 2.99 μV is measured across the width of the strip.

The density of indium is $7.31 \times 10^3 \text{ kg/m}^3$ and its molar mass is 114.818 g/mole; the majority charge carriers in indium are electrons

- Compute the charge carrier concentration (or “number density of charge carriers”) in indium
- Using your answer in (a), compute the Fermi energy of indium, at 293K, in eV
- Compute the average number of charge carriers contributed by each indium atom
- In the diagram below, show the deflection of the charge carriers [Hint: consider the Lorentz force] and indicate the proper sign for the Hall voltage [Hint: use a right-handed coordinate system, and assume that your voltmeter is connected such that voltage increases in the positive y direction] :



4.

You decide to conduct an experiment whereby you want to compare the tensile behaviors of pure Cu in its annealed state, pure Cu with very small grains, and annealed Cu reinforced with oxide particles with an average size of 5 nm and a volume fraction of the particle is 2%.

- Draw the expected stress /strain curves for all three alloys, label all relevant quantities, and describe in details the expected differences and similarities.
- What are the quantitative differences in yield strength at room temperature between the three alloys? State all assumptions made in your calculations.

The inert gas contained in a spherical bubble that is submersed in a liquid slowly resorbs into the surrounding liquid. The rate process involves two steps: adsorption of the gas onto the wall and diffusion of the gas molecules into the liquid. The latter occurs across a boundary layer of thickness $\delta = 0.5$ mm that is governed by the convective flow around the bubble and that is independent of the bubble radius. The system is maintained at ambient conditions, i.e., room temperature and 10^5 Pa. The diffusion coefficient of the gas molecule in the liquid is $D = 10^{-5}$ cm²/s and the molecular weight of the gas is 40g/mol. The adsorption rate is governed by the collision rate of the gas with the bubble wall, and the desorption rate is 20% of the adsorption rate. The maximum solubility of the gas in the liquid is 0.01 mol/cm³ and beyond the boundary layer the convection dilutes the gas to zero concentration.

Fick's second law for spherical coordinates is $\frac{\partial c}{\partial t} = D \frac{1}{r^2} \left(r^2 \frac{\partial^2 c}{\partial r^2} + 2r \frac{\partial c}{\partial r} \right)$. Initially, the gas bubble is

10 mm in diameter

- determine which of the two steps is rate limiting in the resorption of the gas
- Develop the rate equation describing the change in radius of the gas bubble with time, assuming steady-state conditions where applicable.

5.

A nickel-based superalloy aircraft engine turbine blade is manufactured using a casting process that produces an aligned single crystal. The stresses are sufficiently low that the blades are elastically loaded. The preferred orientation along the blade loading axis is one in which the Young's modulus is minimized. This is required to minimize thermal stresses developed during engine-start up and shut down.



Assume:

- $C_{11} = 246.5 \text{ GPa}$
- $C_{12} = 147.3 \text{ GPa}$
- $C_{44} = 124.7 \text{ GPa}$
- $S_{11} = 0.0073 \text{ GPa}^{-1}$
- $S_{12} = -0.0027 \text{ GPa}^{-1}$
- $S_{44} = 0.0080 \text{ GPa}^{-1}$

- a) Using the information provided and your knowledge of elastic anisotropy in single crystals - what crystallographic alignment would you recommend? Support your recommendation with a calculation or calculations.
- b) Calculate the elastic modulus in this direction?
- c) If this stress in the loading axis is assumed to be uniaxial and equal to 100 MPa, calculate the strain in the loading direction.

6.

Assuming a cracked structure is subjected to a constant cyclic stress, $\Delta\sigma$, derive a general, analytical (closed form) expression for the number of cycles to propagate an initial crack, of length a_0 , to final fracture at crack length, a_f . Provide a justification for all assumptions used.

A semi-circular crack of depth $a=0.25\text{cm}$ is discovered on the inner diameter of an aluminum alloy pressure vessel. The crack is oriented along a line parallel to the cylinder axis. The pressure vessel has a length of 100cm; outer diameter of 10cm and an inner diameter of 7cm. If the cylinder is repeatedly pressurized from 0MPa to 75MPa, use the expression in part a) to estimate the cyclic life of the pressure vessel.

The material obeys a Paris-Law relationship with $c=5\times 10^{-15}$ (the units of the relationship are in m/cycle and $\text{MPa}\cdot\sqrt{\text{m}}$) and the crack growth rate has been determined to be proportional to the square of the crack tip plastic zone size.

Show and justify all assumptions. For any stress intensity calculation that may be required, you may assume that $F(a/b)=1.0$ and finite width corrections may be neglected.

The following information may be useful.

The fracture toughness of this aluminum alloy is $30 \text{ MPa}\cdot\sqrt{\text{m}}$;

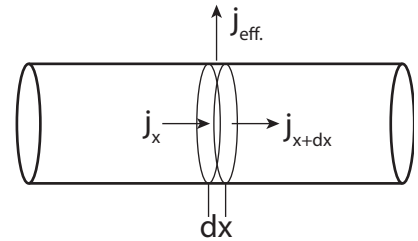
The yield strength of this aluminum is 400MPa ;

$\sigma_{\text{hoop}} = PD/2t$

10.

A hydrocarbon gas containing 1 mol-% N_2 impurities flows through a porous tube at 500 K. The pores in the tube wall are large enough to allow for the N_2 molecules to effuse, but they are too small to let the hydrocarbon molecules through. The total cross section of the pore orifices constitutes 0.01% of the total inside wall surface of the tube. The gas flow rate is 0.5 liters/s

- Derive the expression that gives the N_2 concentration in the gas as a function of the coordinate along the tube axis in flow direction. Given the small concentration of N_2 , ignore any changes in the density of the gas.
- Calculate the length of the tube needed to reduce the N_2 concentration to $1/32$ of its initial value. The molecular weight of $N_2 = 28$ g/mol. The radius of the tube is 1 cm.



12.

The inert gas contained in a spherical bubble that is submersed in a liquid slowly resorbs into the surrounding liquid. The rate process involves two steps: adsorption of the gas onto the wall and diffusion of the gas molecules into the liquid. The latter occurs across a boundary layer of thickness $\delta = 0.5$ mm that is governed by the convective flow around the bubble and that is independent of the bubble radius. The system is maintained at ambient conditions, i.e., room temperature and 10^5 Pa. The diffusion coefficient of the gas molecule in the liquid is $D = 10^{-5}$ cm²/s and the molecular weight of the gas is 40g/mol. The adsorption rate is governed by the collision rate of the gas with the bubble wall, and the desorption rate is 20% of the adsorption rate. The maximum solubility of the gas in the liquid is 0.01 mol/cm³ and beyond the boundary layer the convection dilutes the gas to zero concentration.

Fick's second law for spherical coordinates is $\frac{\partial c}{\partial t} = D \frac{1}{r^2} \left(r^2 \frac{\partial^2 c}{\partial r^2} + 2r \frac{\partial c}{\partial r} \right)$. Initially, the gas bubble is

10 mm in diameter

- c) determine which of the two steps is rate limiting in the resorption of the gas
- d) Develop the rate equation describing the change in radius of the gas bubble with time, assuming steady-state conditions where applicable.