Department of

MATERIALS SCIENCE AND ENGINEERING

Doctoral Written Exam

Core Areas:

Materials Physics And Chemistry
Advanced Mechanical Behavior
Advanced Thermodynamics Of Materials
Kinetics and Phase Transformations
Structure Of Materials

Friday, January 25, 2008

Department of Materials Science and Engineering

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Your exam packet contains 3 questions from each core area for which you signed up, along with several answer sheets. A copy of the Table of Constants is included for your reference. **You must submit 2 questions from each core area you are taking for grading**. You will have 2 hours to complete each section. You can obtain extra answer sheets from the proctor, if needed. Please use the following procedure:

Write a four (4) digit code of your choice, and your name on the 3 X 5 card provided. Use this code in place of your name to identify all answer sheets you submit for both days of the exam. Renee will keep the code information, sealed in an envelope, until after the exams are graded.

For each answer, use the question sheet as the first page of your answer. If additional pages are required, use the blank answer sheets provided. At the end of the examination, staple each question sheet and corresponding answer sheets for each question separately, put this instruction sheet on top of the questions you are turning in, and place them in one side of your exam folder. Place all other exam pages in the other side of your folder, and return everything to Renee if you finish before your time is up.

Please be sure to complete the information required on each page.

CODE NUMBER ________________

CHECK THE QUESTIONS YOU WISH TO HAVE GRADED:

Materials Physics
And Chemistry:

1. _____
2. _____
3. _____

Advanced Mechanical
Behavior:

4. _____
5. _____
6. _____

Advanced Thermodynamics
Of Materials

7. _____
8. _____
9. _____

Kinetics and Phase
Transformations

10.______
11.______
12.______

Structure of Materials

13.______
14.______
15.______

4.

An internal crack has been detected with ultrasonics in the shell of a large cylindrical steel pressure vessel. The stresses in the region of the crack are: longitudinal tensile stress of 60,000 psi; circumferential tensile stress of 140,000 psi; radial compressive stress of 5,000 psi. The pressure vessel is so large that there is no appreciable variation of stress over the region containing the crack. The crack is oriented so that its normal makes an angle of 40° with the longitudinal direction and 50° with the circumferential direction.

(a) What is the normal stress on the plane of the crack? What is the shear stress in the plane of the crack?

(b) If K_{Ic} for the steel is 50 ksi $\sqrt{in}$, what is the smallest size crack that will cause fracture of the pressure vessel? State the assumptions made in arriving at this answer.

5.

A porous Al structure has 30% porosity with uniformly spaced spherical voids.

(a) Determine the length change on each face of a cube, 50 mm^3 in size, made from this material when temperature of sample is raised from RT to 400˚C? (Coefficient of linear thermal expansion of 100% Al is 23 x 10^{-6} /˚C)

(b) If the sample is bonded to a thin strap of invar (thickness = 50 μm) surrounding four adjoining faces of the cube, how much linear expansion do you expect on each face of the cube for the same temperature change?

(Assume: coefficient of linear expansion of invar is 10^{-7} /˚C, elastic modulus and Poisson's ratio of invar = 150 Gpa and 0.3 respectively, elastic modulus and Poissons ratio of Al = 69 Gpa and 0.33 respectively. Assume Al to be ideally plastic with average yield strength of 40Mpa in this temperature range.

6.

A mathematical model of the elastic response of rubber in tension is

$$\sigma = G[(1+e) - 1/(1+e)^2]$$

where e is the engineering strain and G is the shear modulus and σ is the tensile stress.
Also, G = NkT, where N is the number of chain segments per volume, k is Boltzmann's constant and T is the absolute temperature.

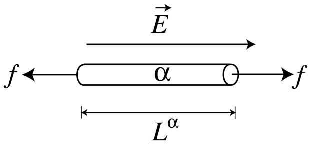
(a) Find expressions for Young's modulus and Poisson's ratio at low strains in terms of G.

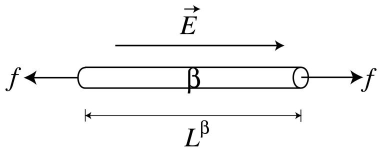
(b) Find an expression for the coefficient of thermal expansion, α, in terms of T and e.

7.

A rod at constant temperature T is stressed at its endpoints with a force f. The stress imposed on the endpoints (f/A_o with A_o the initial cross sectional area at the endpoints of the rod) is much, much larger than the ambient pressure, so that you can neglect the $P \cdot V$ work term in your analysis. An electric field $\vec{E}$ is also imposed along the length of the rod.

The rod is made of a material that can undergo a cubic (α-phase) to tetragonal (β-phase) transformation. At low temperature and low force, the α-phase with length L^α is stable, while at high temperature and large force, the β-phase with length L^β is stable. The length of the β-phase, L^β, is larger than the length of the α-phase, L^α.



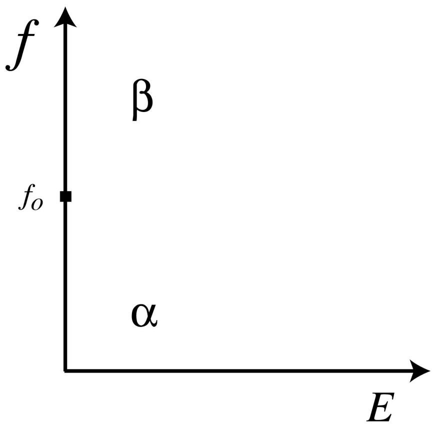


Within an electric field, the two phases exhibit different electric dipole moments $\vec{P}$ according to

$$\vec{P}^\alpha = \chi^\alpha \vec{E} \qquad \vec{P}^\beta = \chi^\beta \vec{E}$$

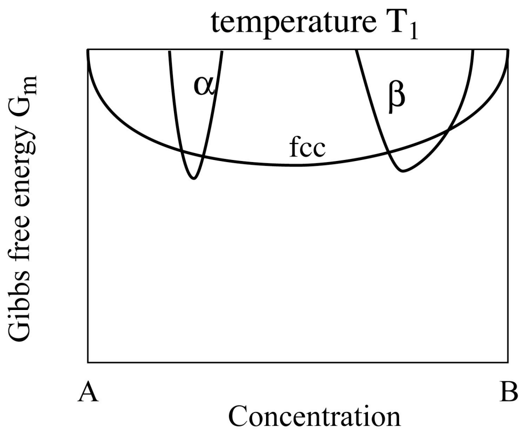
where χ^α and χ^β dielectric susceptibilities with $\chi^\alpha > \chi^\beta$.

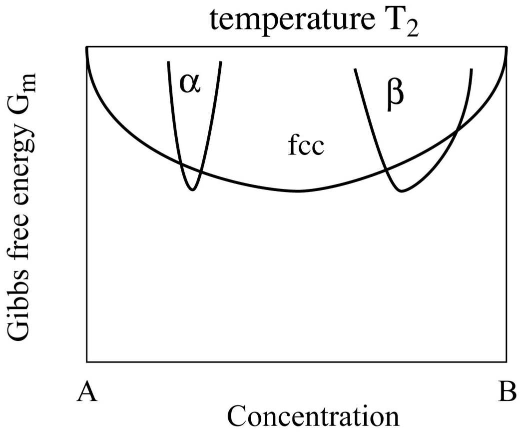
At temperature T and zero electric field, the α → β transformation occurs at a force f_o. How will this transformation force change as the electric field $\vec{E}$ is increased (keeping the temperature constant)? Schematically sketch the transition line between α and β in the f- $\vec{E}$ diagram below. Explain your answer with a thermodynamic analysis.

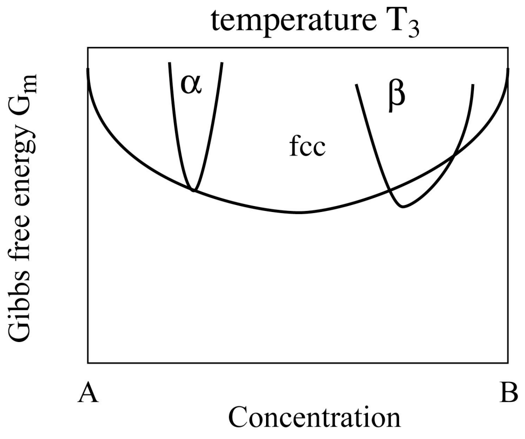


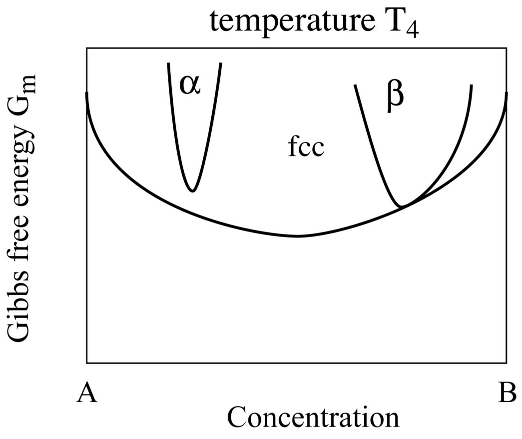
8.

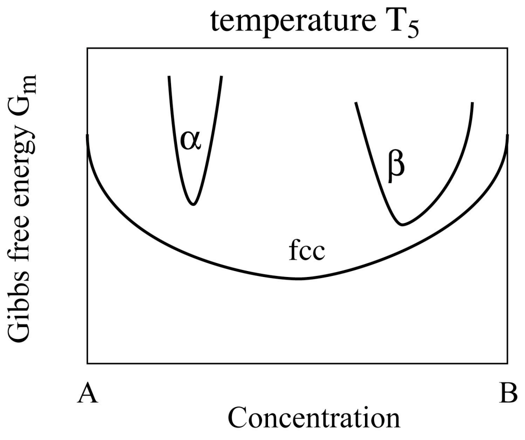
Below are free energy curves of a binary A-B alloy at five different temperatures. Using the blank phase diagrams (see next page), sketch a phase diagram consistent with these free energy curves. Two blank phase diagrams are supplied – one is for practice the other is for your final version. Indicate which phase diagram is your final version.


temperature T_1

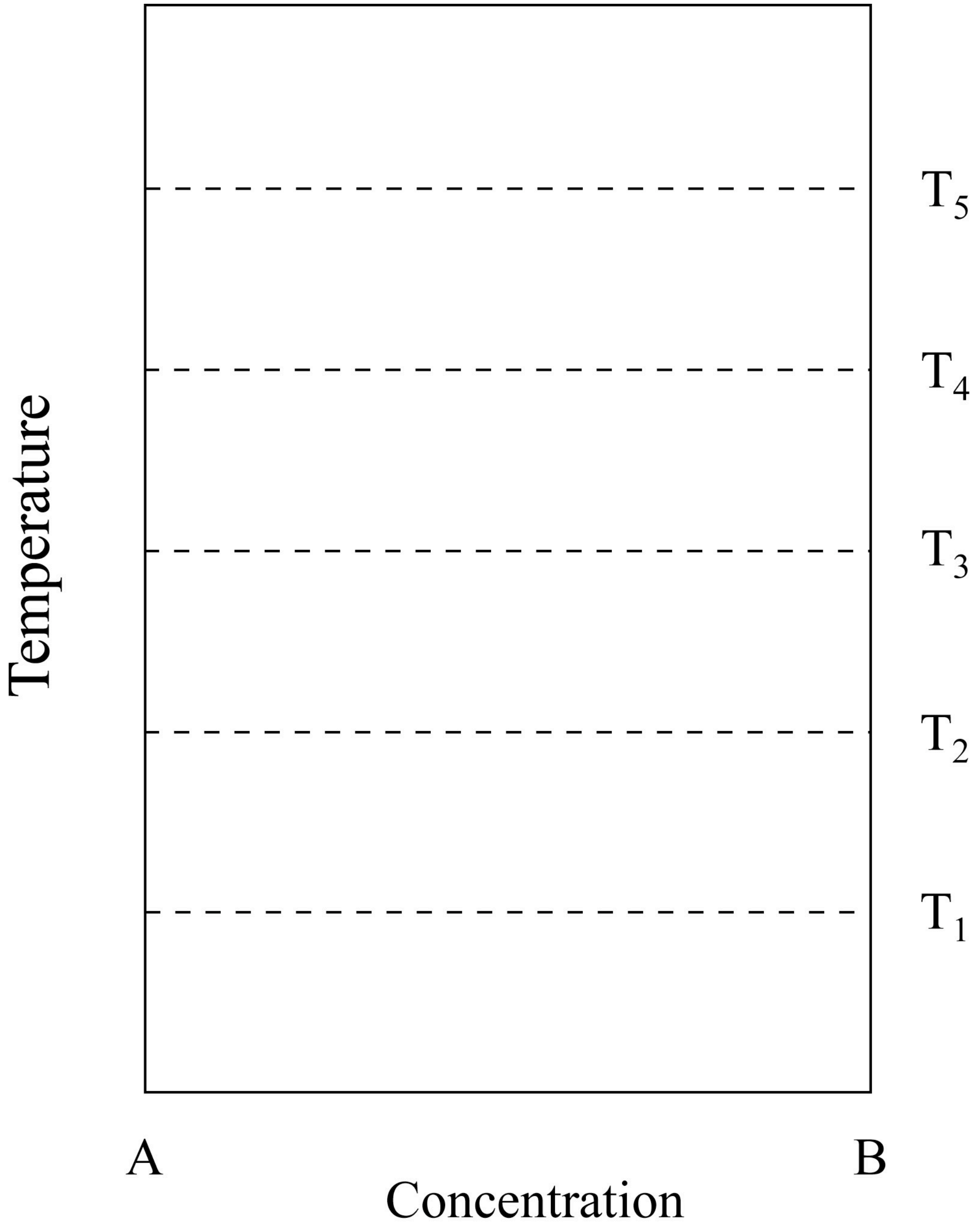

temperature T_2

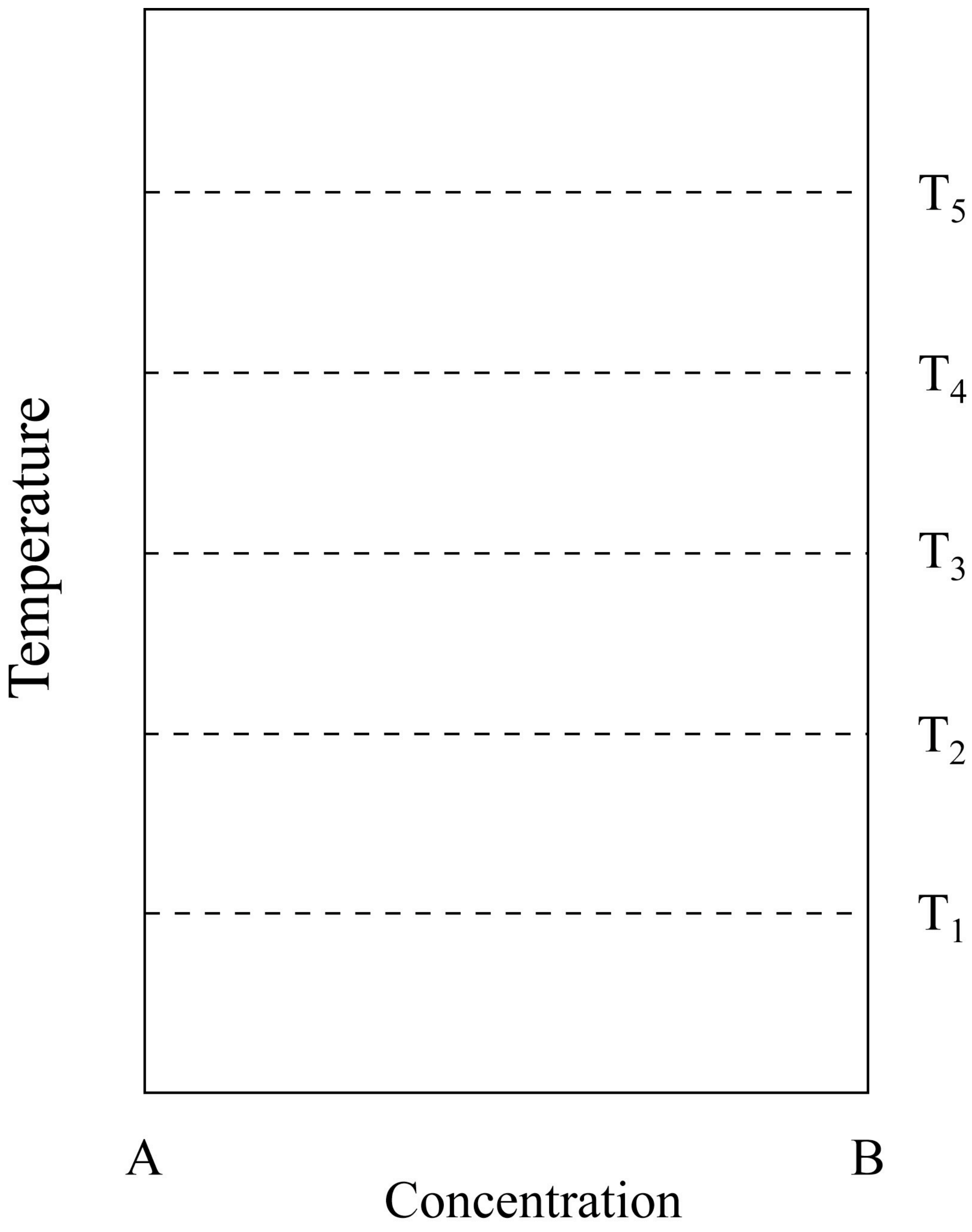

temperature T_3


temperature T_4


temperature T_5

(for practice)




Temperature
T5
T4
T3
T2
T1
A
B
Concentration

9.

Consider the reaction $CO + H_2O = CO_2 + H_2$. All components are gaseous. Initially there are *a* moles of CO, *b* moles of H_2O, *c* moles of CO_2, and *d* moles of H_2 present. The mixture is heated to a temperature *T*, at which the standard Gibbs free energy of the reaction is $\Delta G°$. Calculate the number of moles *x* that have reacted when equilibrium is reached. Assume that $a = 1$ mol; $b = 2$ mol, and $c = d = 0$ mol. Also given are the standard Gibbs free energies of the following reactions:

$CO + \frac{1}{2} O_2 = CO_2$	$\Delta G^\circ_1 = -282400 + 86.81\, T$
$H_2 + \frac{1}{2} O_2 = H_2O$	$\Delta G^\circ_2 = -247500 + 55.85\, T$

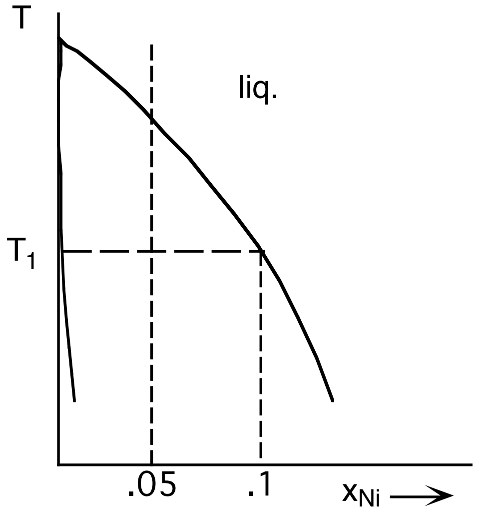
10.

The probability of finding a diffusing particle at a distance *x* after time *t*, is

$$P(x,t)=\frac{1}{2\sqrt{\pi Dt}}\cdot e^{-\frac{x^2}{4Dt}}.$$

Two particles are confined to diffuse within a nano-sized linear tube (i.e., to motion in one-dimensional space). Initially they are appart by a distance *L*. Do you expect that there is a time they are most likely to encounter? If so, what is that time. (Hint: determine first whether there is a most likely location for this encounter to occur, and know that at any location of encounter, the (vector) difference in distance travelled by the two particles is always *L*. Once you know the location for which the probability that both particles are there at the same time maximizes, you can evaluate the time for the particles to get there.)

11.



Consider the solidification of a binary material. A liquid Ni-Ti alloy containing 5 mol% Ni is rapidly cooled and then held at $T_1 = 1300$ °C, as indicated in the adjacent (schematic) phase diagram. At this temperature the solubility of Ni in Ti is negligible, whereas the melt is saturated with 90 mol% Ti. Assume that the solidification front is planar, and progresses strictly perpendicular to the interface. Assume furthermore that the solidification process is entirely controlled by the diffusion of Ni into the melt. Derive a formalism that describes the amount of solidified Ti at early times of the solidification process. (Hint: Begin by establishing the boundary conditions for the diffusion process. You may verify your result by finding that the amount of Ti growth approximately as the square root of time.)

12.

A gas mixture composed of 50 vol% UF_6 and 50 vol% N_2, initially at a pressure of $2 \cdot 10^5$ Pa, is enclosed in a cubic container with permeable walls. The temperature in the container is 300°C. The pores can be considered cylindrical and have an average diameter of 10^{-7} m. There are roughly $1.27 \cdot 10^{10}$ pores per m^2 and the container measures 1m on the side. The gas slowly effuses out of the container through the porous walls. This process is kinetically, not pressure driven.

(a) Explain why the composition of the gas inside the container changes. To this end examine the effusion rate coefficients for each gas.

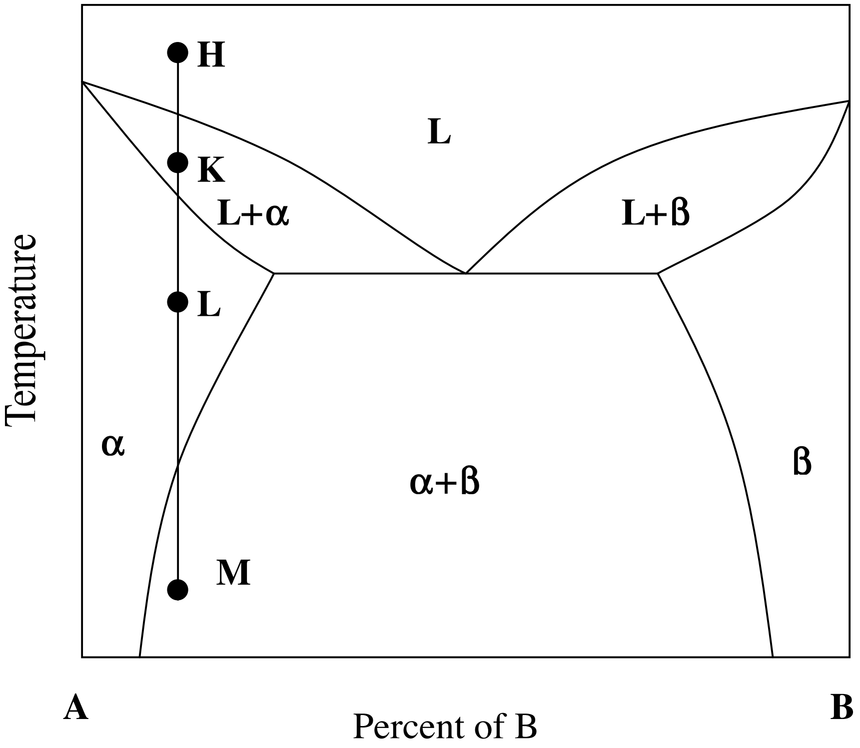
(b) Using the expressions for the effusion rates, derive an equation that describes the drop of pressure as a function of time. Assume that the gas behaves ideally.

(c) What is the maximum difference in concentrations between UF_6 and N_2 that you may expect inside the container?

The molecular weights of UF_6 and N_2 are 352 and 28 g/mol respectively.

13.

Below is a phase diagram of two metals A and B, which are partially soluble. Draw schematics and show the characteristics of powder x-ray diffraction patterns (ϑ–2ϑ) of the solid solutions at H, K, L, and M, respectively. Assuming the α phase has an f.c.c. structure with a = 0.4 nm, while the β phase has a b.c.c structure with a = 0.3nm.



14.

The following schematic electron diffraction pattern of a single, isolated crystallite was obtained on a JEOL 4000 EX TEM operating at 350 kV. It is printed at 1:1 scale. The d-spacing of the planes corresponding to the Bragg spot indicated as spot 1 in the diagram was 0.8 nm.

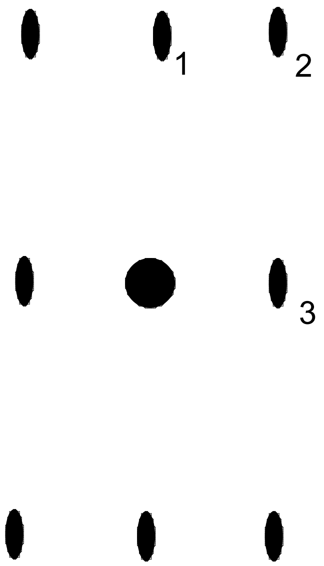
a) What was the wavelength of the electrons?

b) What was the camera length L?

c) What was the camera constant (λ L)?

d) What are the d-spacings of the other two planes (2, 3) observed in the pattern?

e) Draw as accurately as possible the crystallite in real space, including the crystallographic planes corresponding to spots 1 and 3 and the overall size and shape of the crystallite itself.



15.

A tetragonal material has four atoms of the same kind per unit cell located at:

0 1/2 1/4; 1/2 0 1/4; 1/2 0 3/4; 0 1/2 3/4

a. Derive the simplified expressions for F^2.

b. What is the Bravais lattice of this crystal?

c. What are the values of F^2 for the 100, 002, 111, and 011 diffraction maxima?